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Optimal Routing Problem in Dynamic Stochastic Networks

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Abstract

Accidents, bad weather, traffic congestion, etc. contribute to the uncertainties of travel times in real-life transportation networks, which greatly affect the quality of individual life and the reliability of transportation system. In this paper, optimal routing problem is addressed in dynamic transportation networks with random link travel times. Taking the reliability of travel time into consideration, the robust schedule delay is used as the criterion of optimality to evaluate the paths, which is defined as minimizing the largest difference between the actual arriving time and the desired arrival time in dynamic stochastic networks. Under the stochastic consistent condition, a mathematic proof is given to simplify the problem. Then an exact modified Dijkstra's algorithm is designed for finding the optimal routing in STD networks and its computation complexity is calculated as a polynomial-time $o(n^2 * \mathrm{e})$. The validity of the proposed algorithm is also confirmed by conducting a test in a sampled network.

Keuword: Optimal Routing Problem;Schedule Delays; Dynamic Stochastic Networks; Min-Max Approach; Stochastic Consistent Condition

1.Introduction

Accidents, bad weather, traffic congestion, etc. contribute to the uncertainties of travel timesin real-life transportation networks. To an individual traveller, the variant of travel time reduces the quality of life by consuming leisure time, increasing anxiety, and wasting personal resources. To firms, it reduces the productivity of employees and increases freight transportation costs. In order to compensate for the random disruptions to the travel time, the uncertainties should be captured in transportation networks, where all link travel times are treated as dynamic random variables. This paper is to address the optimal routing problem in time-dependent and stochastic (STD) networks with a priori traffic conditions information provided by Advanced Traveller Information System (ATIS) or historical experience.

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According to the fourth comprehensive transport survey in Shanghai, 49% of the travel demand in this city is for commuting purpose and the proportion of business travel such as go to meetings or plane-catching is also increasing rapidly (General Report of the Fourth Comprehensive Transport Survey in Shanghai, 2010). When faced with uncertainty, these travellers with rigid arrival time requirements, are mostly concerned about the reliability of their travel times or the schedule delay. For example, unreliable travel times will cause anxiety or cause disutility to travellers because of unexpected later or earlier arrival at their destinations. So the criteria of optimal routing are extended to reliability measures, such as travel time variance and early/late schedule delays (Gao, 2005).

In this paper, we refer to the schedule delay as the criterion of optimality to evaluate the travel time reliability, which is defined as the difference between the actual arrival time and the desired arrival time. For commuters, late arrival at the workplace would cause trouble, whereas, arrival too early means a waste of time, so arriving around the work starting time in the morning might be the best desire. Therefore in this paper, both early schedule delay and late schedule delay will be considered, allowing different penalties to be associated with each. At a certain departing time,since the actual arriving time at workplace is still a random variablein STD networks, so the schedule delays are still uncertain, which can be represented as a range. Through robust approach, we seek to find a more reliable path whose largest schedule delay over all the feasible paths is minimal, so a routing choice with the most robust travel time schedule delayis viewed as the optimal.

The paper is organized as follows. In Section 2, a literature review is given on optimal path problems under different assumptions of the network. In Section 3, the STD network and model of the optimal path are defined. In section 4, a modified Dijikstra's algorithm is present and the validity of the algorithm is confirmed. Conclusions are made and future directions are given in Section 5.

2. Literature Review

In STD networks, the information that traveller needed to make decisions can be classified as a priori information or online information. A priori information is about the general picture of the day-to-day fluctuations of traffic quantities, e.g. the travel time on a link is 2 minutes on average, but roughly once in a week, the travel time is unusually high, due to various reasons. Online information is about the traffic condition on a specific day, e.g. an incident just occurred on this link, and it will probably last for 10 to 20 minutes (Gao& Chabini, 2002, 2006). In this literature review, the focus is on the optimal path in STD networks with a priori information provided by ATIS or historical experience.

Hall (1986) first put forward the stochastic and time-dependent shortest path problem (STDSPP). He showed that standard shortest path algorithms (such as Dijkstra's algorithm) do notfind the minimum expected cost path on a non-stationary stochastic network and that the optimalroute choice is not a simple path but a policy. He chose minimum expected travel time (METT) as the optimality criterion and dynamicprogramming was proposed for finding the METT path in STD networks. Miller-Hooksand Mahmassani (1998) showed and compared algorithms for the least possible cost path in thediscrete-time non-stationary stochastic case. Miller-Hooks and Mahmassani (2000) explored the definition of optimality based on first-order stochastic dominance and definite dominance and an algorithm for finding the least expected cost pathin that case was provided.

A number of works addressed the STD optimal path finding problem by using the theory of stochastic processes. Psaraftis and Tsitsiklis (1993) considered a problem in which the travel time distributions on links evolve over time according toa Markov process except that the changes in the status of the link are not observed until the vehicle arrives at the link. Azaron and Kianfar (2003) extended the research of Psaraftis and Tsitsiklis(1993) to the real situation problems. They altered the assumption that the environmental variables evolve in accordance with the independent semi-Markov processes instead of Markov processes. The length of each arc is assumed an exponential random variable. Upon arriving at each node, travellers know the state of the environmental variable of this node and also those of its forward nodes.

After the 1990s, the utility theory in economics had been introduced to solve the STD optimal path problem. For the non-stationary stochastic case, Wellman et al. (1995) developeda revised path-planning algorithm. He identified a stochastic consistent condition that justified a generalized dynamic-programming approach with stochastic dominance. He presented a revised path-planning algorithm based on utility function where a stochastic consistency condition holds. He and Gao (2012) defined a disutility function of travel time to evaluate the STD paths. They designed an exact label-correcting algorithm to find the optimal path with the minimum expected disutility, based on a new property for which Bellman's principle holds. But the algorithm had exponential worst-case computation complexity.

In recent years, considering the "worst-case performance" of each path, the robust optimization theory has emerged as a pre-emption way to address the uncertainties of link travel times. Dimitris and Melvyn (2002) proposed a linear robust optimization approach based on polyhedral uncertainty sets. Melvyn (2004) proposed a new methodology that promises greater computational tractability, both theoretically and practically than the classical robust framework. He then proved that the robust optimal path problem in stochastic networks can be solved by solving a deterministic shortest path problem without extra complexity. But he didn't conduct a further study to find out if the new methodology and the simplifying process are also adapted to solving the problem in the STD networks.

In summary, the above review shows that a large body of approaches has emerged in the past decades addressing the problem, but most of the approaches to this problem generally need a precise probability distribution of the uncertain link travel times which is hard to realize in practical application and high computation complexity and inefficient algorithms are still strong restraints when solving large size networks problems. In recent years, the robust optimization theory has emerged as a preemption way to cope with the uncertainties.

3. Problem Statement and Optimality Equation

3.1. The stochastic time-dependent network

Given a directed graph $G=(N,A,T,C_{ij}^{t})$,*N* is set of the nodes and *A* is set of the links. The number of the nodes and links are denoted respectively as $|N|=n$ and $|A|=m$. *S* represents the set of links with random travel time. In this paper, it is assumed that all link travel times in the network are random and dynamic, so $|s|=|A|=m$. *T* is the set of time periods $\{0,1,...,K-1\}$. Link *(i, j)* represents the directional link from node *i* to node *j*. A path between two distinguished nodes can be denoted as a sequence of consecutive nodes. The travel time on link *(i, j)*at time period *t* is denoted as C_{ij}^{t} . Waiting is not permitted at the previous node *i* before moving forward to the next node. We define $C_{ij}^{t}=R_{ij}^{t}+\tau_{ij}^{t}$, where R_{ij}^{t} is a fixed value at a certain time period *t* and τ_{ij}^{t} is a random variable ranges from 0 to d_{ij}^{t} . So C_{ij}^{t} takes value in $\left[R_{ij}^{t},R_{ij}^{t}+d_{ij}^{t}\right]$, where d_{ij}^{t} is also a fixed value at time period *t* as well as R_{ij}^{t} . Link travel times with entry times between time period 0 and K−2 are random and time-dependent, while those at and beyond K−1 are static and deterministic. The time period between 0 and K−2 can be viewed as the peak hour period in real transport networks, when the travel times have higher variability than those in off-peak hours, which is represented by the time period at and beyond K−1.

In addition, the assumption is that the network satisfies the *Stochastic Consistent Condition* proposed by Wellman et al. (1995). That is, the network is stochastically consistent, if for any link *(i, j)*, at any time period $t<t'$ and any given time z, the following inequality holds.

$$\Pr(C_{ij}^{t}+t\le z)\ge \Pr(C_{ij}^{t'}+t'\le z) \qquad (1)$$

This appears to be the most naturalgeneralization of the deterministic consistency condition (*FIFO Property*). It means that the probability of arriving by any given time z cannot be increased by leaving later. This assumption is in accordance with the transportation network in real world, that is, there exists overtaking phenomenon in urban streets, but in general the probability of the earlier-leaving vehicle arriving by any given time z is more than the later-leaving one.

3.2. Optimality equation for Min-Max schedule delay problem in STD networks

In this paper, the routing with the most robust travel time schedule delay is viewed as the optimal path. Under the Min-Max criterion, we seek to find a more reliable path whose largest schedule delay over all the feasible paths is minimal. Both early schedule delay and late schedule delay will be considered, allowing different penalties to be associated with each. For commuters or plane-catchingpeople, late schedule delay may cause more troubles, so any possibility of arriving late is not permitted and given an infinite penalty in this paper.

Let X be the set of feasible solutions of the problem. Path $\lambda(\lambda \in X)$ denotes any candidate path between the origin node *O* and the destination node *D*. $Cost(o,\lambda,t)$ is definedas the travel time of Path $\lambda(\lambda \in X)$ when departing from the origin node *O* at time period *t*. $Max\ Cost(o,\lambda,t)$ represents the largest travel time of Path $\lambda(\lambda \in X)$, whereas, $Min\ Cost(o,\lambda,t)$ means the opposite. Let *B* be a big enough positive number. Time t^* denotes the desired arrival time or the work starting time. The mathematic formulation is given as follows for optimal path finding:

$$Z = \underset{\lambda \in X}{Min}\ Max\left[B \bullet \max\left(0, Cost(o,\lambda,t) - t^*\right) + \max\left(0, t^* - Cost(o,\lambda,t)\right)\right] \quad (2)$$

Rewritten as:

$$Z = \underset{\lambda \in X}{Min}\left[B \bullet \max\left(0, Max\ Cost(o,\lambda,t) - t^*\right) + \max\left(0, t^* - Min\ Cost(o,\lambda,t)\right)\right] \quad (3)$$

In the above equation, we seek to minimize the largest schedule delays, including the late schedule delays and the early schedule delays for finding the optimal routing. The penaltyof early arriving is set as 1 and arriving late is not permitted. From the definitions in 3.1, the Min-Max optimization model is equivalent to:

$$Z = Min\left[B \bullet \max\left(0, \underset{(i,j)\in A}{Max}\sum_{i,j\in S}\sum_{t=1}^{T}\left(R_{ij}^t + \tau_{ij}^t\right) \bullet x_{ij}^t - t^*\right) + \max\left(0, t^* - \underset{(i,j)\in A}{Min}\sum_{i,j\in S}\sum_{t=1}^{T}\left(R_{ij}^t + \tau_{ij}^t\right) \bullet x_{ij}^t\right)\right] \quad (4)$$

S.t

$x_{ij}^t = 1$ when the link (i, j) is occupied at time period t, otherwise $x_{ij}^t = 0$

$$\sum_{i,j\in A}\sum_{t=1}^{T} x_{ij}^t = 1$$

$$\sum_{t=1}^{T}\sum_{\{j:(i,j)\in A\}} x_{ij}^t - \sum_{t=1}^{T}\sum_{\{j:(j,i)\in A\}} x_{ji}^t = \begin{cases} 1, & i = node\ O \\ -1, & i = node\ D \\ 0, & others \end{cases}$$

Proposition 1. Under the stochastic consistent condition, the optimal routing problem in STD networks can be simplified into solving a minimum problem in specific time-dependent networks.

Proof. As mentioned in Eq. (1), the network in this study satisfies the stochastic consistent condition, so for any link *(i, j)*, at any time period $t < t'$ and any given time z, the following inequality holds:

$$\Pr(C_{ij}^t + t \le z) \ge \Pr(C_{ij}^{t'} + t' \le z)$$

Because $C_{ij}^t = R_{ij}^t + \tau_{ij}^t$, the inequality can be rewritten as follow:

$$\Pr\,(R_{ij}^{t}+\tau_{ij}^{t}+t\le z)\ge \Pr\,(R_{ij}^{t'}+\tau_{ij}^{t'}+t^{'}\le z)\ (5)$$

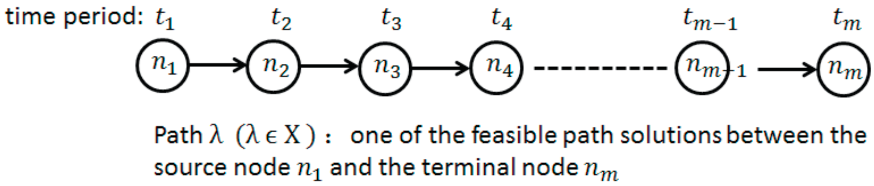

Fig. 1. A graphical representation of Path $\lambda\left(\lambda \grave{o}\, X\right)$

Step 1: Path $\lambda\left(\lambda \grave{o}\, X\right)$ shown in Fig. 1is defined as any feasible pathbetween the origin node n_1 and the destination node n_m. It can be denoted as a sequence of consecutive nodes: $\left\{n_1, n_2 \ldots n_m\right\}$. Assuming that the traveller departs from the origin node n_1 at time period t_1, $\mathrm{t}_1 \in \left\{0,1,\ldots,K-1\right\}$ and arrives at the node n_2 at time period t_2 .The link travel time between the origin node n_1 and the next node n_2 is $C_{12}^{t_1}$,so $t_2 = C_{12}^{t_1}+t_1$.Because $C_{12}^{t_1}$ takes values in $\left[R_{ij}^{t}, R_{ij}^{t}+d_{ij}^{t}\right]$, we can have followings:

$$R_{12}^{t_1}\le C_{12}^{t_1}\le R_{12}^{t_1}+d_{12}^{t_1}\ (6)$$

$$t_2 = C_{12}^{t_1}+t_1, R_{12}^{t_1}+t_1\le t_2\le R_{12}^{t_1}+d_{12}^{t_1}+t_1\ (7)$$

We define $max\,tt_2 = R_{12}^{t_1}+d_{12}^{t_1}+t_1, min\,tt_2 = R_{12}^{t_1}+t_1$. For any departure time period t_1, the inequality $min\,tt_2\le t_2\le max\,tt_2$ holds.

Step 2: As we have defined,there is no waiting time at the previous node before moving forward to the next, so we assume that the traveller departs from node n_2 at time period t_2 and arrives at the node n_3 at time period t_3. The link travel time between node n_2 and node n_3 is $C_{23}^{t_2}$,so $t_3 = C_{23}^{t_2}+t_2$.

According to Eq. (1),we know that the probability of the earlier-leaving vehicle arriving by any given time z is more than the later-leaving one.

(**Step 2a**) Because $t_2\le \max\, tt_2$, so we can have the following inequality:

$$\Pr\,(C_{23}^{t_2}+t_2\le z)\ge \Pr\,(C_{23}^{max\,tt_2}+\max tt_2\le z)\ (8)$$

Let $Z = max(C_{23}^{max\,tt_2}+\max tt_2)$, the inequality is equivalent to：

$$Pr\left(C_{23}^{t_2}+t_2\le max(C_{23}^{max\,tt_2}+\max tt_2)\right)\ge Pr\left(C_{23}^{max\,tt_2}+\max tt_2\le max(C_{23}^{max\,tt_2}+\max tt_2)\right)\ (9)$$

It could be easily found that the probability of the right-side formula is 100%, so we can have:

$$\Pr\left(C_{23}^{t_2}+t_2\le max\left(C_{23}^{max\,tt_2}+\max tt_2\right)\right)\ge 1 \qquad (10)$$

Then we can determine that if Eq. (10)always holds, it must have the following relation:

$$C_{23}^{t_2}+t_2\le max(C_{23}^{t_2}+t_2)\le max(C_{23}^{max\,tt_2}+\max tt_2)\ (11)$$

That is,

$$t_3 = C_{23}^{t_2}+t_2\le max\, C_{23}^{max\,tt_2}+\max tt_2 = R_{23}^{max\,tt_2}+d_{23}^{max\,tt_2}+\max tt_2 = R_{23}^{max\,tt_2}+d_{23}^{max\,tt_2}+R_{12}^{t_1}+d_{12}^{t_1}+t_1\ (12)$$

(**Step 2b**) Besides, because $t_2\ge \min\, tt_2$, according to Eq. (1), we can have the following inequality:

$$Pr\left(C_{23}^{min\,tt_2} + min\,tt_2 \le z\right) \ge Pr\left(C_{23}^{t_2} + t_2 \le z\right) \quad (13)$$

Let $Z < min\left(C_{23}^{min\,tt_2} + min\,tt_2\right)$, the inequality is equivalent to：

$$Pr\left(C_{23}^{min\,tt_2} + min\,tt_2 < min\left(C_{23}^{min\,tt_2} + min\,tt_2\right)\right) \ge Pr\left(C_{23}^{t_2} + t_2 < min\left(C_{23}^{min\,tt_2} + min\,tt_2\right)\right) \quad (14)$$

It could be easily found that the probability of the left-side formula is 0%, so we can have:

$$Pr\left(C_{23}^{t_2} + t_2 < min\left(C_{23}^{min\,tt_2} + min\,tt_2\right)\right) = 0 \quad (15)$$

Then we can determine that if Eq. (15)always holds, it must have the following relation:

$$C_{23}^{t_2} + t_2 \ge min\left(C_{23}^{min\,tt_2} + min\,tt_2\right) \quad (16)$$

That is,

$$t_3 = C_{23}^{t_2} + t_2 \ge min\left(C_{23}^{min\,tt_2} + min\,tt_2\right) = R_{23}^{min\,tt_2} + min\,tt_2 = R_{23}^{min\,tt_2} + R_{12}^{t_1} + t_1 \quad (17)$$

Let $max\,tt_3 = R_{23}^{max\,tt_2} + d_{23}^{max\,tt_2} + R_{12}^{t_1} + d_{12}^{t_1} + t_1, min\,tt_3 = R_{23}^{min\,tt_2} + R_{12}^{t_1} + t_1$ .For any departure time period t_1, the inequality $min\,tt_3 \le t_3 \le max\,tt_3$ holds.

Then **Step 3**, **Step 4** until **Step m-1**, the traveller arrives at the destination node n_m .

Step m-1: Assuming that the travellerarrives at the destination node n_m at time period $\mathrm{t_n}$, we can have the following two recursion formulas:

Recursion formula1:

$$t_{\mathrm{n}} = \mathrm{C}_{(\mathrm{n-1})\mathrm{n}}^{\mathrm{t_{n-1}}} + t_{\mathrm{n-1}} \le \max \mathrm{C}_{(\mathrm{n-1})\mathrm{n}}^{\max tt_{\mathrm{n-1}}} + \max \mathrm{tt_{n-1}} = \mathrm{R}_{(\mathrm{n-1})\mathrm{n}}^{\max tt_{\mathrm{n-1}}} + \mathrm{d}_{(\mathrm{n-1})\mathrm{n}}^{\max tt_{\mathrm{n-1}}} + \cdots\cdots + \mathrm{R}_{23}^{\max tt_2} + \mathrm{d}_{23}^{\max tt_2} + \mathrm{R}_{12}^{\mathrm{t_1}} + \mathrm{d}_{12}^{\mathrm{t_1}} + \mathrm{t_1} \quad (18)$$

Recursion formula2:

$$\mathrm{t_n} = \mathrm{C}_{\mathrm{n-1n}}^{\mathrm{t_{n-1}}} + \mathrm{t_{n-1}} \ge \min C_{(\mathrm{n-1})n}^{\min tt_{\mathrm{n-1}}} + \min \mathrm{tt_{n-1}} = \mathrm{R}_{(\mathrm{n-1})\mathrm{n}}^{\min tt_{\mathrm{n-1}}} + \cdots\cdots + \mathrm{R}_{23}^{\min tt_2} + \mathrm{R}_{12}^{\mathrm{t_1}} + \mathrm{t_1} \quad (19)$$

The recursion mentioned above is equivalent to the following inequality:

$$t_n - t_1 \le \mathrm{R}_{(\mathrm{n-1})\mathrm{n}}^{\max tt_{\mathrm{n-1}}} + \mathrm{d}_{(\mathrm{n-1})\mathrm{n}}^{\max tt_{\mathrm{n-1}}} + \cdots\cdots + \mathrm{R}_{23}^{\max tt_2} + \mathrm{d}_{23}^{\max tt_2} + \mathrm{R}_{12}^{\mathrm{t_1}} + \mathrm{d}_{12}^{\mathrm{t_1}} = \sum_{i,j\in A}\sum_{t=1}^{T}\left(R_{ij}^{t} + d_{ij}^{t}\right)\bullet x_{ij}^{t} \quad (20)$$

$$t_n - t_1 \ge \mathrm{R}_{(\mathrm{n-1})\mathrm{n}}^{\min tt_{\mathrm{n-1}}} + \cdots\cdots + \mathrm{R}_{23}^{\min tt_2} + \mathrm{R}_{12}^{\mathrm{t_1}} + \mathrm{t_1} = \sum_{i,j\in A}\sum_{t'=1}^{T} R_{ij}^{t'} x_{ij}^{t'} \quad (21)$$

x_{ij}^{t} and $x_{ij}^{t} = 1$ when the link (i, j) is occupied at time period t, otherwise x_{ij}^{t} and $x_{ij}^{t} = 0$

For any departure time period $\mathrm{t_1}$, the above inequality always holds, so we can obtain that:

$$Travel\ Time = t_n - t_1 \quad (22)$$

$$\sum_{i,j\in A}\sum_{t'=1}^{T} R_{ij}^{t'} x_{ij}^{t'} \le t_n - t_1 \le \sum_{i,j\in A}\sum_{t=1}^{T}\left(R_{ij}^{t} + d_{ij}^{t}\right)\bullet x_{ij}^{t} \quad (23)$$

$$Max\left(t_n - t_1\right) = \sum_{i,j\in A}\sum_{t=1}^{T}\left(R_{ij}^{t} + d_{ij}^{t}\right)\bullet x_{ij}^{t} \quad (24)$$

$$Min\left(t_n - t_1\right) = \sum_{i,j\grave{o}A}\sum_{t'=1}^{T} R_{ij}^{t'} x_{ij}^{t'} \tag{25}$$

$Max\left(t_n - t_1\right)$ represents the worst case performance (the highest travel time) of the feasible Path λ and it is equivalent to $Max\ Cost\left(o,\lambda,t\right)$, so we can have:

$$Max\ Cost\left(o,\lambda,t\right) = \sum_{i,j\grave{o}A}\sum_{t=1}^{T}(R_{ij}^{t} + d_{ij}^{t}) \bullet x_{ij}^{t} \tag{26}$$

$Min\left(t_n - t_1\right)$ represents the optimal case performance (the lowest travel time) of the feasible Path λ and it is equivalent to $Min\ Cost\left(o,\lambda,t\right)$, so we can have:

$$Min\ Cost\left(o,\lambda,t\right) = \sum_{i,j\grave{o}A}\sum_{t'=1}^{T} R_{ij}^{t'} x_{ij}^{t'} \tag{27}$$

So the Min-Max optimization model (Eq. 4) can be rewritten as follows:

$$Z = Min\ \left[B \bullet \max\left(0, \sum_{i,j\grave{o}S}\sum_{t=1}^{T}\left(R_{ij}^{t} + d_{ij}^{t}\right) \bullet x_{ij}^{t} - t^{*}\right) + \max\left(0, t^{*} - \sum_{i,j\grave{o}A}\sum_{t'=1}^{T} R_{ij}^{t'} x_{ij}^{t'}\right)\right] \tag{28}$$

S.t

x_{ij}^{t} and $x_{ij}^{t} = 1$ when the link (i, j) is occupied at time period t, otherwise x_{ij}^{t} and $x_{ij}^{t} = 0$

$$\sum_{i,j\grave{o}A}\sum_{t=1}^{T} x_{ij}^{t} = 1 \text{ and } \sum_{i,j\grave{o}A}\sum_{t'=1}^{T} x_{ij}^{t'} = 1$$

$$\sum_{t=1}^{T}\sum_{\{j:(i,j)\grave{o}A\}} x_{ij}^{t} - \sum_{t=1}^{T}\sum_{\{j:(j,i)\grave{o}A\}} x_{ji}^{t} = \begin{cases} 1, & i = node\ O \\ -1, & i = node\ D \\ 0, & others \end{cases}$$

$$\sum_{t'=1}^{T}\sum_{\{j:(i,j)\grave{o}A\}} x_{ij}^{t'} - \sum_{t'=1}^{T}\sum_{\{j:(j,i)\grave{o}A\}} x_{ji}^{t'} = \begin{cases} 1, & i = node\ O \\ -1, & i = node\ D \\ 0, & others \end{cases}$$

In this model, $\mathrm{R}_{ij}^{t} + \mathrm{d}_{ij}^{t}$ and $\mathrm{R}_{ij}^{t'}$ just change along with the time period t. So under the stochastic consistent condition, the STD optimal path problem has already beensimplifiedinto solving a minimum problem in a specifictime-dependent network.

4. The Modified Dijkstra's Algorithm

In real world, circuits will generally not exist in travellers' routes,which are also not permitted in this paper. A modifiedDijkstra's algorithm here is designed to find the optimal routingin STD networks. Thealgorithm's computation complexity is $\mathrm{o}\left(n^2 * \mathrm{e}\right)$, where *n* and *e* are respectivelythe numbers of the nodes and links of the network.

4.1. Notations and variables

$\boldsymbol{t}_0$ —Departure time

$\boldsymbol{v}_{\boldsymbol{o}}$ —Origin vertex (node), v_d —destination vertex (node)

v —Set of all vertices (nodes)

A—Set of all links

S—Set of the visited nodes

W— Set of the unvisited nodes

$C_{ij}(t)$—Linktravel time of link (i, j) at time period t. A bounded dynamic random variable

$C_{ij}^{max}(t)$—The maximumtravel time of link (i, j) at time period t

$C_{ij}^{min}(t)$—The minimumtravel time of link (i, j) at time period t

l_i^{max}—Tentative maximum travel times at the vertex i

l_i^{min}—Tentative minimum travel times at the vertex i

l_i—Label of the vertex i (the largest deviation of the vertex i) and $l_i = t^* - l_i^{min}$

pre_i—Predecessor vertex (node) of the vertex i (node)

P_i—Set of all the predecessor nodes of node i

4.2. Algorithm statement

For a given originvertex (node) in the graph, the algorithm can find the path with lowest cost (i.e. the shortest length path) between it and every other vertex (node). It can also be used to find the lowest cost paths from a single origin vertex to a single destination vertex by stopping the algorithm when the optimal path to the destination vertex has been determined.

The node from which we are starting is defined as the origin node. The modified algorithmwill assign some initial distance values (travel time deviation) and try to update them step by step.

Step 1: Initialization

Assign to every node a tentative distance value (travel time deviation): set it to zero for our initial node and to infinity for all other nodes. Mark all nodes unvisited. Set the origin node as current node. Createa set *W*and a set P_i .Set *W*is called the unvisited set consisting of all the unvisited nodes except the source node. Set P_i is the set of all the predecessor nodes of node i.

Let $i = v_0, l_i^{max} = 0, l_i^{min} = 0, l_i = 0, pre_i = 0, P_i = \{0\}, \textit{ for } \forall j \neq i, l_j = +\infty, \mathrm{S} = \{i\}, W = \phi$

Step 2: Label Comparison

For the current node i, consider all of its neighbors except the nodes in P_i and calculate their tentative worst-case distances (largest travel times) l_i^{max} and the optimum-case distances (optimum travel times) l_i^{min} . If the largest early schedule delays $l_j = t^* - l_j^{min}$ is less than the previously recorded tentative largest deviation, then overwrite that label. Even though a neighbor has been examined or marked as visited, it remains in the unvisited set at this time.

$\forall j \grave{o} V_i = V - P_i, l_j^{max} = t_o + l_i^{max} + \mathrm{C}_{ij}^{max}\left(t_o + l_i^{max}\right), l_j^{min} = t_o + l_i^{min} + \mathrm{C}_{ij}^{min}\left(t_o + l_i^{min}\right)$

A. $\textit{if } l_j^{max} > t^*$ then, $l_j = +\infty$

B. $\textit{if } l_j^{max} < t^* \textit{and } l_j > t^* - l_j^{min}$ then, $l_j = t^* - l_j^{min}, pre_j = i, P_j = P_i \cup \{i\}, \mathrm{S} = \mathrm{S} - \{j\}, W = W \cup \{j\}$

Step 3: Update Labels

Set the node v* marked with the smallest tentative largest travel time deviation as the next "current node".

$l_{v*} = min\ l_j, j \in W, S = S \cup \{v*\}, W = W - \{v*\}, i = v*$

Step 4: Stop and Find the Optimal Path

If the set *W*is empty and labels of all the nodes can't be decreased more in step 2, then stop. Otherwise go back to Step 2 to continue the path finding.We can trace the way back in the set P_D consisting of all the predecessor nodes of the destination node D, following the arrows in reverse to find the optimal path from the origin node to the destination node.But if the label of the destination node stay infinite and can't be decreased more, it means taking any one of the feasible paths to the destination will probably miss the desired arrival time. In other words, no matter which route to take, it can't guarantee the commuters arriving at the workplace on time.

4.3. Computation complexity of the algorithm

As the literature review shows in Section 2, the approaches to STD optimal path problem generally lead to a great increase in computation complexity, which is an issue when solving large size network problems. So the computation complexity of the proposed algorithm need to be analyzed for implementation purpose.As the algorithm statement shown above, the computation complexity of the algorithm can be calculated as $\mathrm{o}\left(n^2 * \mathrm{e}\right)$, so it retains a relatively low polynomial-time complexity.

4.4. The validity of the modified algorithm

We conduct a small computational test on a sampled STD traffic network (shown in Figure 2) in order to show the validity of the proposed modified label setting algorithm. The traffic network is sampled with treating link travel times stochastic and time-dependent for every link and every time period. The modified Dijkstra's algorithm will be conducted to find the optimal path between the origin Aand the destination D at time period 1. The computational process and results are shown in Table 1.

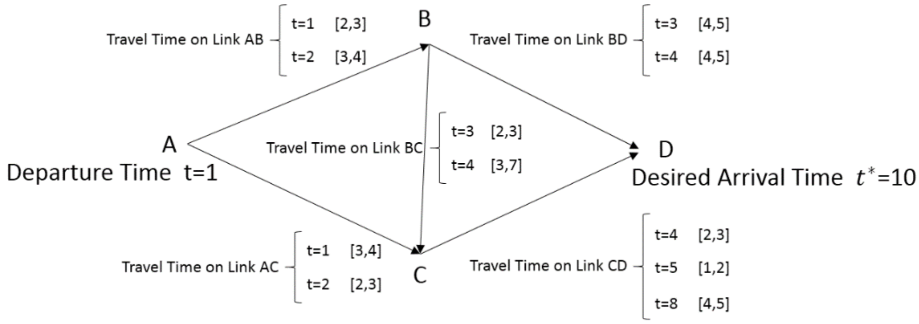

Fig.2. The sampled STDnetwork

Table 1.Computational process table

Node i		A	B	C	D
$s_1(i)$	l_i^{min}	0	0	0	0
	l_i^{max}	0	0	0	0
	l_i	0	$+\infty$	$+\infty$	$+\infty$
	P_i	0	0	0	0
$s_2(i)$	l_i^{min}		3	4	0
	l_i^{max}		4<10	5<10	0

	l_i		7	6^*	$+\infty$
	P_i		A	A	0
$s_3(i)$	l_i^{min}		3		6
	l_i^{max}		4<10		7<10
	l_i		7^*		4
	P_i		A		A, C
$s_4(i)$	l_i^{min}			5	7
	l_i^{max}			11>10	9
	l_i			$+\infty$	1
	P_i			A, B	A, B
$s_5(i)$	l_i^{min}	0	3	4	7
	l_i^{max}	0	4<10	5<10	9
	l_i	0	7^*	6^*	1
	P_i	0	A	A	A, B

-----The dash area in $s_4(i)$ is to show that any possibility of arriving late is not permitted

From the computational process table, we can tracethe way back in set P_D, following the arrows in reverse to find the optimal path from the origin node to the destination node. So the optimal path in the sampled STD network is the Route A-B-D with the most robust schedule delays.

In order to show the validity of the modified algorithm, we can enumerate all the feasible routes from Node A to Node D and calculate the schedule delay. The schedule delays range can be listed as follows:

Table 2.Schedule Delays Range Table

Route	Actual Arriving Time Range	Schedule Delays Range
Route A-B-D	[7,9]	[1,3]
Route A-C-D	[6,7]	[3,4]
Route A-B-C-D	[6,t] t>desired arrival time	Arrivinglate is not permitted

It can be easily found from Table 2 that Route A-B-D has a more reliable schedule delay range, where the largest schedule delays over all the feasible routes is minimal. Although Route A-C-D can also guarantee the travellers arriving at the destination not being late, it may waste more time at some possibility. From the result of this small computational test, we can conclude that based on the Min-Max criterion, the proposed approach and the modified Dijkstra's algorithm can solve for the optimal routing problem in STD networks. Although the complexity of the algorithm retains polynomial-time, in the future more tests should be conducted on large size networksto examine the applicability and efficiency of the approach.

5. Conclusion and Future Direction

This paper addresses the optimal routing problem in stochastic time-dependent networks through Min-Max approach. In order to optimally route the travellers with rigid arrival time requirements, who are mostly concerned about the reliability of their travel times,the robust schedule delays is used as the criterion of optimality to evaluate the travel time reliability.Under the stochastic consistent condition, a mathematic proof is given to simplify the optimal equation. An exact modified Dijkstra's algorithm is designed for finding the optimal routing in STD networks. The computation complexity of the algorithm is polynomial-time $o(n^2 * e)$. The validity of the proposed algorithm is also confirmed by conducting a small test in a sampled network.

There usually exist strong dependencies among random link travel times, so some extended works will be continued on analyzing the characteristics of stochastic distributions and dependencies among link travel times. More computational tests should be conducted on large size networksin the future to examine the applicability and efficiency of the approach. By using the real-data, the validity of the stochastic consistent condition proposed by Wellman also should be confirmed in the near future work.

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